K

middle layer to the back layer to give an upper value for the exposure time t_E to the heat flux that gives a large temperature differential through the thickness. If the top layer were thermally isolated, equation (6) gives the temperature rise ΔT for the exposure time t_E with an input heating power W to the top layer of thickness $\Delta h/3$ as:

(14)
$$\Delta T = 3W t_E / \rho c_D \Delta h$$

We now estimate the heat transferred through the middle layer, thickness $\Delta h/3$, to the back layer during the exposure time t_E . We use equation (15) to estimate the temperature difference across the middle layer as $\Delta T/4$ (e.g., $\sim \Delta T/2$ for the top layer, $\sim 1/2(\Delta T/2)$ for both middle and bottom layers. Equation (13) then gives for the heat transfer through the middle layer to the back layer as:

(area) W = k (area) (
$$\Delta T/4$$
) / ($\Delta h/3$)

(15)
$$W = (3/4)k\Delta T/\Delta h$$

Substituting for ΔT from equation (14) and solving for t_E gives the estimate for the upper bound to the exposure time for the regime with a large temperature differential as:

(16)
$$t_E < (4/9)\rho c_p \Delta h^2 / k$$

We note that this expression does not explicitly contain the input heating power; however since the heat capacity c_p and the thermal conductivity k are somewhat temperature dependent for all materials there is an implicit dependence on the heat input by way of the temperature dependence of these variables.